Continuing Education in Engineering
The College of Engineering
University of California Extension

WATER RESOURCES ENGINEERING EDUCATIONAL SERIES

Program VII

POLLUTION OF COASTAL AND ESTUARINE WATERS

San Francisco, California
January 28-30, 1970

TABLE OF CONTENTS

Faculty Member in Charge

DAVID K. TODD, Professor of Civil Engineering, University of California, Berkeley.

Instructional Staff

NORMAN H. BROOKS, Professor of Civil Engineering, California Institute of Technology, Pasadena.

HUGO B. FISCHER, Assistant Professor of Civil Engineering, University of California, Berkeley.

JEROME B. GILBERT, Executive Officer, Water Resources Control Board, State of California, Sacramento.

HARVEY F. LUDWIG, President, Engineering-Science, Inc., Arcadia, California.

JOHN D. PARKHURST, Chief Engineer and General Manager, County Sanitation Districts of Los Angeles County, Los Angeles.

ERMAN A. PEARSON, Professor of Sanitary Engineering and Chairman, Division of Hydraulic and Sanitary Engineering, University of California, Berkeley.

DONALD W. PRITCHARD, Director, Chesapeake Bay Institute, Johns Hopkins University, Baltimore, Maryland.

ROBERT E. SELLECK, Associate Professor of Sanitary Engineering, University of California, Berkeley.

PHILIP N. STORRS, Vice-President, Engineering-Science, Inc., Oakland.

Section 8

CONCEPTUAL DESIGN OF SUBMARINE OUTFALLS - I
JET DIFFUSION

by

Norman H. Brooks*

I. Introduction

Effective utilization of the ocean for natural purification of sewage effluent depends on good jet mixing and oceanic convection and diffusion. Important factors are ocean currents, density stratification, bacterial dieoff, and the use of multiple-jet diffusers.

A. General Plan. The degree of treatment and the design of an outfall are interrelated as a system and depend on what receiving water standards must be maintained. The most feasible system in the ocean is usually a primary treatment plant connected to a deep-water outfall terminating with a multiple-port diffuser to produce high initial dilution. Chlorination (if any) may be intermittent.

In estuaries, however, where overall flushing may be the limiting factor, special diffusers for high initial dilution may be of no special benefit. Hence this chapter deals primarily with discharges to the ocean with ample currents.

B. Oceanography. Initial planning for an ocean outfall should include oceanographic surveys in the vicinity of possible discharge sites to determine:

---

*Professor of Civil Engineering, W. M. Keck Laboratory of Hydraulics and Water Resources, California Institute of Technology, Pasadena, California.

1. Currents (direction, magnitude, frequency, variation with depth, relation to tides, water displacements)
2. Densities (variation with depth determined from salinity and temperature data and standard tables (41)**)
3. Submarine topography, geology, and bottom materials
4. Marine biology
5. Turbidity
6. Dissolved oxygen, etc.

The final site selection for an ocean outfall is usually based on general characteristics of the coastal waters and on topography of the drainage area. Details of diffuser design are developed after the general site is chosen.

C. Summary of Design Problems for an Ocean Outfall.

1. Design for diffusion. Since the objective of an ocean outfall is to disperse sewage effluent in the ocean, first consideration is given to the analysis of diffusion and dieoff of sewage organisms. From such an analysis one establishes a satisfactory length of outfall from shore, a desirable diffuser pipe arrangement (length and orientation), and the approximate number and spacing of ports.

There are essentially three stages of turbulent mixing or diffusion of sewage discharged into the ocean:

1. Initial jet mixing (considering jet strength, currents, and density differences).
2. Development of a homogeneous sewage field.
3. Turbulent diffusion of sewage field as a whole due to natural oceanic turbulence.

**Numbers refer to the reference list at the end.

Methods of analysis of stage 1 will be presented in detail below. There is no special analysis for stage 2 above, as it is only a brief transition period between stages 1 and 3. Natural oceanic diffusion (stage 3) will be covered in a later lecture.

2. Hydraulic design. The second stage of the design is then the hydraulic design of the outfall pipe and diffuser based on a manifold analysis. The discharge will not distribute itself uniformly among the ports unless the port diameters are carefully varied to balance the effects of friction, change of pipe depth, and variability of the discharge coefficient for discharge through side ports in a pipe with flow.

D. New Developments in Ocean Outfalls. (For a semi-technical summary, see Brooks (32, 33).)

1. Multiple-port diffusers to greatly increase dilution.
2. Utilization of natural stable density stratification in the ocean to keep sewage field submerged and to prevent contamination of surface water.

II. Diffusion of a Single Buoyant Jet without Density Stratification.

(References: Rawn, Bowerman, and Brooks (28), Abraham (2, 3), and Fan and Brooks (14, 15).) (See Reference List for additional items.)

A. Dilution Graphs. The dilution S_o at the top of a rising plume formed by a horizontal jet (Fig. 1) is a function of the following variables (assuming no temperature gradients or currents):

y_o = depth from surface to center of discharge jet

D = diameter of jet at point of discharge

Q = jet discharge (of sewage effluent)

$g' = \frac{\Delta\rho}{\rho} g$ = apparent acceleration due to gravity where $\frac{\Delta\rho}{\rho}$ is the relative density difference, commonly about 0.026.

The viscosity does not enter directly, because Reynolds numbers are so large that the flow is fully turbulent.

The four variables may be arranged into two dimensionless groups, namely

y_o/D, and

$$\mathbb{F} = \frac{Q}{\frac{\pi}{4}D^2\sqrt{g'D}} = \frac{V}{\sqrt{g'D}} = \text{Froude number} \tag{1}$$

where V = nominal mean velocity of discharge. Therefore, the dilution S_o is a function

$$S_o = f(y_o/D, \mathbb{F}). \tag{2}$$

The functional relation can best be shown graphically as in Figs. 2 and 3. Fig. 2, from Rawn, Bowerman, and Brooks (28), is based entirely on experiments by Rawn and Palmer (27), whereas Fig. 3 from Fan and Brooks (14) covers a wider range of values, and is based on theoretical computer solutions. The theoretical analysis by Fan and Brooks (14) is almost identical to Abraham's (3), but is simpler and more readily extended to stratified environments.

For ordinates $2\sqrt{2}\,\alpha\, y/D$ (= $0.23y/D$ for $\alpha = 0.082$) greater than 50 (off the top of Fig. 3), the following asymptotic form for simple plumes in uniform environment may be used for the centerline dilution:

$$S_o = .092(y/D)^{5/3}\mathbb{F}^{-2/3} = \frac{0.078g'^{2/3}y^{5/3}}{Q^{2/3}} \qquad (3)$$

Since $\mathbb{F} \sim D^{-5/2}$ for given Q, S_o is actually independent of D in this regime.

A comprehensive collection of numerical solutions to buoyant jet problems has been presented in graphical form by Fan and Brooks (15) including both round and two-dimensional turbulent jets in either uniform or linearly-stratified environments. Results include the centerline dilutions, width of the plumes, and trajectories, for various angles of discharge ranging from horizontal to vertical.

The trajectory and jet widths are shown in Fig. 4 for horizontal round jets (15). In the ordinate and abscissa it is recommended that the *entrainment* coefficient $\alpha = 0.082$, or $2\sqrt{2}\alpha = 0.23$. The half-width w is considered to be the 2σ (i.e. two standard deviations of concentration distribution curve across the plume).

B. <u>Example.</u> Consider discharge of Q = 50 mgd of sewage effluent from a diffuser at a depth of 65 feet in homogeneous sea water (no stratification). There is sufficient head to jet the sewage out at 15 fps. Sea water specific gravity is 1.025 and sewage is 0.999. Compare initial dilutions which can be obtained by a 50-port diffuser with a 5-port one. Assume ports are rounded inside and produce no jet contraction, and are separated sufficiently to avoid interference.

Required total area of jets $= \dfrac{50 \times 1.55 \text{ cfs}}{15} = 5.17 \text{ ft}^2$

a. For 50 ports:

$$\frac{\pi}{4} D^2 = \frac{5.17}{50} = 0.1034 \text{ ft}^2$$

$$D = 0.362 \text{ ft.} = 4.35 \text{ in.}$$

$$y_o / D = 65/0.362 = 180$$

$$F = \frac{V}{\sqrt{\frac{\Delta\rho}{\rho} gD}} = \frac{15}{\sqrt{(.026)(32.2)(.362)}} = 27$$

By Fig. 3, $S_o = 95$

b. For 5 ports:

$$\frac{\pi}{4} D^2 = \frac{5.17}{5} = 1.034 \text{ ft}^2$$

$$D = 1.15 \text{ ft} = 13.75 \text{ in.}$$

$$y_o / D = 65/1.15 = 57$$

$$F = \frac{15}{\sqrt{(0.26)(32.2)(1.15)}} = 15.2$$

By Fig. 2, $S_o = 35$, or by Fig. 3, $S_o = 29$.

(The difference is within the experimental error.)

Notes: (1) All solutions assume no interference between rising plumes. For case (a) (50 ports), Fig. 4 gives $w/D = 32$ at $y_o/D = 180$ for $F = 27$; therefore, the diameter at the head of the plume is approximately $2w = 2 \times 32\ (0.36) = 23$ ft. Thus, to avoid interference, there should be at least 23 ft between jets on the same side of the pipe. Similarly, for case (b) we find $w/D = 17$ for $y_o/D = 57$ and $F = 15$; therefore, the diameter of top at plume is predicted to be about $2w = 2 \times 17 \times 1.15 = 39$ ft.

(2) If a lower discharge velocity were used, the dilutions S_o would be less for both cases, as seen from the figures.

None of the above calulations should be considered highly accurate, as errors may be of the order of $\pm$ 10 to 20%, due to turbulent fluctuations, density stratification, and currents in the ambient fluid. However, qualitatively it is very clear that using a large number of small jets increases jet dilution.

The largest number of ports ever used on an ocean outfall (as far as is known to the author) is 742, ranging in diameter from 2.0 to 3.6 in., on the fourth ocean outfall of The County Sanitation Districts of Los Angeles (completed in 1965). The overall length of this 120-inch outfall is 11,880 feet, including 4440 feet of diffuser pipe at depths of 165 to 190 feet.

For diffusers with many jets in a row, the effect is that of a line source or slot. For full details on slot jets, the reader is referred to (15). For large depths the asymptotic formula for two-dimensional plumes in uniform environment is:

$$S_o = 0.38 \frac{g'^{1/3} y}{q^{2/3}} \qquad (4)$$

wherein q is the discharge per unit length of diffuser pipe, and S_o is the centerline dilution.

III. Limiting Height of Rise of a Buoyant Plume in a Stratified Environment

A. Problem and Notation. The purpose of this section is to show how, in an elementary way, to predict whether or not a sewage field will be submerged below the surface of the ocean when there is stable density stratification. The problem to be solved is shown in Fig. 5; it may be considered either as axisymmetric (point source case) or two-dimensional (line source case). Notation is as follows:

Q_o = discharge from point source

q_o = discharge per unit length from line source

ρ_d = density of fluid released at source

ρ_1 = density of ambient fluid at level of source

ρ_o = density of ambient fluid at height y above source

$\frac{d\rho_o}{dy}$ = density gradient

g = acceleration due to gravity

y_{max} = maximum height of rise of plume

The formulas for the case of the point source are based on the analysis given by Morton, Taylor, and Turner (21); using the same general procedure, Brooks and Koh (10) extended the work for line sources.

B. Assumptions. The following simplifying assumptions are made:

1. Source is a simple point source, or line source.
2. Fluid is released from source with zero initial momentum.*
3. Variations in ρ are small compared to ρ.

---

*Brooks and Koh (10) have shown by analyses including initial momentum as an added parameter that this is a reasonable assumption for outfall problems. See also Fan and Brooks (15).

4. Density gradient of the ambient fluid is constant (i. e., linear density profile).

5. Plume is turbulent and rate of mixing around edges of plume at height y is proportional to a characteristic velocity and size at that height.

6. Profiles of density deficiency and velocity are geometrically similar at all heights.

C. Analysis. The analysis is based on three fundamental equations: continuity of volume flux, continuity of buoyancy flux, and momentum. Using the similarity assumption the authors integrate over the cross-sectional area of the plume at every height y and obtain three simultaneous ordinary differential equations for the three dependent variables:

$y(y)$ = upward velocity on the centerline of the plume

$\rho(y)$ = density on the centerline of the plume

$b(y)$ = nominal radius of the plume

The numerical integration by the authors yields graphs of these functions in dimensionless form.

D. Application to Rising Column of Sewage Effluent (or Digested Sludge) in Sea Water. Of special interest is the height of rise of a plume, found by setting the velocity of rise equal to zero, i.e.

$$u(y_{max}) = 0$$

After the plume reaches this height it sinks back to a slightly lower position where its density is in balance with the ambient fluid (approximately at 0.8 y_{max}). Because of the momentum of the plume during its rise it "overshoots" this neutral position.

1. Point sources. From the equations of Morton, Taylor, and Turner (21) and an experimentally determined entrainment constant ($\alpha = 0.093$) it may be shown that:

$$y_{max}^{4} = 198 \frac{Q_o\sqrt{\rho_1}(\rho_1 - \rho_d)}{\sqrt{g}\left|\frac{d\rho_o}{dy}\right|^{3/2}} \tag{5}$$

Oceanographers frequently use σ (or σ_t) for specific gravity (S.G.) of ocean water, defined as

$$\sigma = (S.G. - 1) \cdot 1000 \qquad (gr/ml)$$

$$\text{or } S.G. = 1 + \frac{\sigma}{1000} \qquad \text{(at atmospheric pressure)}$$

Thus if $\sigma = 25.0$, it means S.G. = 1.0250. Using σ units equation (5) reduces to

$$y_{max}^{4} = 198 \frac{Q_o(1000 + \sigma_1)^{1/2}(\sigma_1 - \sigma_d)}{\sqrt{g}\left|\frac{d\sigma_o}{dy}\right|^{3/2}} \qquad (5a)$$

Since the variation in the term $(1000 + \sigma_1)^{1/2}$ is very slight (and one also must take the fourth root to find y_{max}), a single average value of σ_1 may be used, such as $\sigma_1 = 25$. Substituting this value, equation (5a) becomes

$$y_{max}^{4} = 6340 \frac{Q_o(\sigma_1 - \sigma_d)}{\sqrt{g}\left|\frac{d\sigma_o}{dy}\right|^{3/2}} \qquad (5b)$$

Note that the σ's and the constant 6340 are dimensionless; any consistent set of units may be used for y, Q, and g (such as ft, cfs, and ft/sec^2).

The analysis gives the following value for the terminal dilution (centerline or minimum value):

$$S_t = 0.28 \frac{g^{1/8}(\sigma_1 - \sigma_d)^{3/4}}{Q_o^{1/4}\left|\frac{d\sigma_o}{dy}\right|^{5/8}} \qquad (6)$$

Convenient tables of sea water density as a function of temperature and salinity have been published by the U.S. Navy Hydrographic Office (41).

2. Line sources. Following the same method of analysis and approximation, the maximum height of rise for a line source (6) is

$$y^3_{max} = 610 \frac{q_o(\sigma_1 - \sigma_d)}{\sqrt{g}\left|\frac{d\sigma_o}{dy}\right|^{3/2}} \tag{7}$$

The corresponding terminal dilution is given by

$$S_t = 0.41 \frac{g^{1/6}(\sigma_1 - \sigma_d)^{2/3}}{q_o^{1/3}\left|\frac{d\sigma_o}{dy}\right|^{1/2}} \tag{8}$$

E. Examples

1. Point sources. Consider a discharge of 50 mgd from a multiple port diffuser at 100 feet depth when there is a density gradient in the ocean as follows:

Seawater, bottom (-100 ft):	σ_o = 25.80 (S. G. = 1.02580)*
Seawater, top:	σ_o = 24.60 (S. G. = 1.02460)*
Effluent:	σ_d = -0.50 (S. G. = 0.99950)

Determine whether the sewage field will be submerged or not for the case of (a) 50 ports, and (b) 5 ports. For approximate analysis on safe side, neglect initial horizontal momentum and extra mixing it induces near the bottom. Also assume no interference between jets.

$$\text{Gradient, } \frac{d\sigma_o}{dy} = \frac{1.20}{100} = 0.0120 \text{ per ft}$$

For (a) 50 ports, by equation (5b):

$$Q_o = \frac{50 \times 1.55}{50} = 1.55 \text{ cfs/port}$$

$$y^4_{max} = 6340 \frac{(1.55)(25.8 + 0.5)}{\sqrt{32.2}(0.0120)^{3/2}} = 0.346 \times 10^8$$

$$y_{max} = 77 \text{ ft (which is less than the total depth = 100 ft)}$$

The field will be submerged according to the analysis.

---

*Equivalent to temperature difference of ~10°F.

The terminal dilution by equation (6) is

$$S_t = \frac{0.28\ (32.2)^{1/8}(25.8 + 0.5)^{3/4}}{(1.55)^{1/4}\ (0.0120)^{5/8}} = 72$$

For (b), 5 ports

$$Q_o = 15.5 \text{ cfs/port}$$

$$y_{max} = 77 \text{ ft} \sqrt[4]{\frac{15.5}{1.55}} = 137 \text{ ft} = \text{depth (if available)}$$

required to stop rise of plume under given gradient.(Note that $y_{max} \geq$ depth = 100 ft).

Thus it is predicted that sewage will rise to the surface.

For the example, it is apparent that multiple-port diffusers greatly enhance the possibility of generating a submerged sewage field.

2. Line source. Suppose in case (a) above that there is interference between the 50 individual rising plumes because they are spaced only 5 feet apart on each side of the pipe. Since the initial jets are circular and horizontal, there is a brief phase of circular jet mixing before the individual jets merge to produce an effective line source as shown in Figure 6. During this phase, density stratification is not important, and, using the results of Albertson et al. (8), it may be predicted that the jets would interfere after only about 12 feet of travel. The buoyancy flux term $q(\sigma_1 - \sigma_d)$ does not change during this phase (because $(\sigma_1 - \sigma_d)$ decreases proportionately to the increase in q). Calculations are as follows for the maximum height of rise:

$$q_o = \frac{1.55 \text{ cfs/port}}{5 \text{ feet/port}} = 0.31 \text{ cfs/ft}$$

Equation (6) gives (with previous data):

$$y^3_{max} = 610 \frac{(0.31)\ (25.8 + 0.5)}{\sqrt{32.2}\ (0.0120)^{3/2}} = 0.66 \times 10^6 \text{ft}^3$$

$$y_{max} = 87 \text{ ft (still less than total depth of 100 ft).}$$

But to this value must be added the height of the effective line source above the port level as depicted in Figure 6 or about 3 feet according to Figure 4. Thus, considering interference the predicted height of rise is increased only from 77 to 90 ft.

The corresponding terminal dilution is found by equation (8):

$$S_t = 0.41 \frac{(32.2)^{1/6} (25.8 + 0.5)^{2/3}}{(0.31)^{1/3} (0.0120)^{1/2}} = 88$$

For a detailed example of the use of these formulas for predicting dilution and submergence, refer to a recent report on sludge disposal in Puget Sound by Municipality of Metropolitan Seattle (37).

F. <u>Practical Application of Buoyant Plume Formulas to Non-Linear Stratification.</u> Frequently the density gradient in the ocean is not constant as presumed by equations (5b), (6), (7) and (8). In that case a reasonable approximation is obtained by assuming an equivalent uniform gradient over that part of the total depth through which the sewage plume rises. The following approximate method of application has been developed by the writer, and used in the design of the Orange County Outfall, California (40):

In Figure 7, the point of discharge is O and ρ_o is the density of environment. Between O and A, the mean gradient is

$$-\left.\frac{d\sigma}{dy}\right|_{ave} = \frac{\Delta\sigma_a}{y_a}$$

For simplicity $\Delta\sigma$ is defined as positive for a stable profile.

Substituting into equation (5b) for <u>point sources</u> we get

$$y^4_{max} = 6340 \frac{Q_o(\sigma_1 - \sigma_d)}{\sqrt{g}\left(\frac{\Delta\sigma_a}{y_a}\right)^{3/2}}$$

Now let $y_a = y_{max}$, so that we use the mean gradient over the full height of rise. Then we get

$$y_{max}^{5/2} = 6340 \frac{Q_o(\sigma_1 - \sigma_d)}{\sqrt{g}} (\Delta\sigma_a)^{-3/2}$$

$$y_{max} = \left[6340 \frac{Q_o(\sigma_1 - \sigma_d)}{\sqrt{g}}\right]^{2/5} (\Delta\sigma_a)^{-3/5} \tag{9}$$

Similarly for line sources, equation (7) becomes

$$y_{max} = \left[610 \frac{q_o(\sigma_1 - \sigma_d)}{\sqrt{g}}\right]^{2/3} (\Delta\sigma_a)^{-1} \tag{10}$$

Equations (9) and (10) may now be plotted on a graph of y_{max} vs. $\Delta\sigma$ for given Q_o (or $q_o = Q_o/s$ where s = port spacing) (see Fig. 8), and overlain on observed density profiles. Note that for high values of $\Delta\sigma$ the point source solution gives higher values of y_{max}, whereas for small $\Delta\sigma$ the line source solution gives the higher values of y_{max}. It is recommended that in either case the higher value of the solution y_{max} always be used as a best estimate. For large heights of rise, the flow pattern over a multiple-port diffuser will become essentially a line plume, whereas for low rises the diffuser may generate essentially a series of independent round plumes.

For the three ambient profiles in Fig. 8, the solutions are shown as y_1, y_2, y_3. By using a transparency for the curves representing equations (9) and (10), it is easy to investigate different possible depths of discharge at many times of year (varying density profiles).

This procedure is believed to be conservative, i.e., to over-estimate the values of y_{max}. For more accurate analyses, one should use a computer program which accommodates arbitrary density gradients and allows for initial jet momentum. Such a program has been prepared for round jets by Ditmars (12) and is available on request.

G. Limitations of Buoyant Plume Formulas. Besides the non-linearity of density gradients described in Sec. F above, some other limitations are: (1) that the discharge jet is not vertical with no momentum, but usually horizontal with appreciable initial lateral momentum; (2) that ocean currents have not been included in the analysis; and (3) that no allowance has been made for major changes in the overall environment due to waste discharges. However, limited field checks show that equation (5b) works in spite of its apparent limitations.

Recent research by Fan (13, 15) at Caltech presents analytical solutions for horizontal buoyant jets in a stratified environment without current. With horizontal jet discharge the height of rise is reduced somewhat below that indicated for simple plumes; hence using the simple plume formula is on the conservative side. Also treated by Fan (13) is the case of a vertical jet into a uniform current, which shows that even very weak currents can bend over a rising plume and carry it long distances downstream before it rises to the surface. The case of a buoyant jet in a stratified current has not yet been solved analytically or experimentally.

Other recent references are listed on the following pages.

Acknowledgments

These notes have been prepared for the Water Resources Engineering Educational Series, Program VII, "Pollution of Coastal and Estuarine Waters," University Extension and the College of Engineering, University of California, Berkeley, Jan. 28-30, 1970.

Some of the research results given here have been developed at the W. M. Keck Laboratory of Hydraulics and Water Resources, California Institute of Technology, with support of the Federal Water Pollution Control Administration, Dept. of Interior, Grant No. 16070 DGY, and predecessor grants.

REFERENCE LIST

A. Jet Diffusion, Including Effects of Density Difference.

1. Abraham, G., "Jet Diffusion in Liquid of Greater Density," Trans. ASCE, Vol. 126, Part I, 1961, pp. 76-88.
2. Abraham, G., "Jet Diffusion in Stagnant Ambient Fluid," Delft Hydraulics Laboratory Publication No. 29, July 1963.
3. Abraham, G., "Horizontal Jets in Stagnant Fluid of Other Density," J. Hyd. Div., ASCE, Vol. 91, No. HY4, July 1965, pp. 139-154. (Closing discussion, Vol. 93, No. HY1, January 1967, pp. 63-68.)
4. Abraham, G., "Entrainment Principle and Its Restrictions to Solve Problems of Jets," Journal of Hydraulic Research, Vol. 3, No. 2, pp. 1-23, 1965.
5. Abraham, G., and Brolsma, A. A., "Diffusers for Disposal of Sewage in Shallow Tidal Water," Delft Hydraulics Laboratory Publication No. 37, Oct. 1965 (also Proc., XI Congress, International Assoc. for Hydraulic Res., Leningrad, Sept. 1965, Paper 2.1).
6. Abraham, G., and Eysink, W. D., "Jets Issuing into Fluid with a Density Gradient," Journal of Hydraulic Research, Vol. 7, No. 2, 1969, pp. 145-175.
7. Anwar, H. O., "Behavior of Buoyant Jet in Calm Fluid," J. Hyd. Div., ASCE, Vol. 95, No. HY4, July 1969, pp. 1289-1303.
8. Albertson, M. L., Dai, Y.B., Jensen, R.A., and Rouse, H., "Diffusion of Submerged Jets," Trans. ASCE, Vol. 115, 1950, pp. 639-697.
9. Bosanquet, C. H., Horn, G., and Thring, M.W., "The Effect of Density Differences on the Paths of Jets," Proc. Roy. Soc. of London A, 263, 1961, pp. 340-352.
10. Brooks, N. H., and Koh, R.C.Y., "Discharge of Sewage Effluent from a Line Source into a Stratified Ocean," International Assoc. for Hydraulic Res., Proc. XI Congress, Leningrad, Sept. 1965, Paper 2.19.
11. Cederwall, Klas, "Hydraulics of Marine Waste Water Disposal," Hydraulic Division, Chalmers Institute of Technology, Goteborg, Sweden, Report No. 47, January 1968.
12. Ditmars, J. D., "Computer Program for Round Buoyant Jets into Stratified Ambient Environments," W.M. Keck Laboratory of Hydraulics and Water Resources, Tech. Memo 69-1, California Inst. of Tech., Pasadena, March 1969.

13. Fan, Loh-Nien, "Turbulent Buoyant Jets into Stratified or Flowing Ambient Fluids," W. M. Keck Laboratory of Hydraulics and Water Resources Report No. KH-R-15, Calif. Inst. of Tech., Pasadena, Calif., June 1967.

14. Fan, L. N., and Brooks, N. H., Discussion of Reference 3 ("Horizontal Jets in Stagnant Fluid of Other Density"), J. Hyd. Div., ASCE, Vol. 92, No. HY2, March 1966, pp. 423-429.

15. Fan, L. N., and Brooks, N. H., "Numerical Solutions of Turbulent Buoyant Jet Problems," W. M. Keck Laboratory of Hydraulics and Water Resources Report No. KH-R-18, California Inst. of Tech., January 1969.

16. Frankel, R. J., and Cumming, J. D., "Turbulent Mixing Phenomena of Ocean Outfalls," J. of the Sanitary Engrg. Div., ASCE, Vol. 91, No. SA2, April 1965, pp. 33-59.

17. Hart, W. E., "Jet Discharge into a Fluid with a Density Gradient," J. Hyd. Div., ASCE, Vol. 87, No. HY6, November 1961, pp. 171-200.

18. Hino, Mikio, "Forced Plumes in a Stably Stratified Fluid," Trans. Japan Soc. of Civil Eng., No. 86, Oct. 1962, pp. 29-37 (in English).

19. Hume, N. B., Bargman, R. D., Gunnerson, C. G., and Imel, C. E., "Operation of a 7-Mile Digested Sludge Outfall," Trans., ASCE, Vol. 126, Part III, 1961, pp. 306-331.

20. Lee, Shao-Lin and Emmons, H. W., "A Study of Natural Convection above a Line Fire," Journal of Fluid Mechanics, Vol. 11, 1961, pp. 353-368.

21. Morton, B. R., Taylor, G. I., and Turner, J. S., "Turbulent Gravitational Convection from Maintained and Instantaneous Sources," Proc. Roy. Soc. A, Vol. 234, Jan.-Mar. 1956, p. 1-23.

22. Morton, B. R., "Forced Plumes," Journal of Fluid Mechanics, Vol. 5, 1959, pp. 151-163.

23. Morton, B.R., "On a Momentum-Mass Flux Diagram for Turbulent Jets, Plumes, and Wakes," Journal of Fluid Mechanics, Vol. 10, 1961, pp. 101-112.

24. Murgai, M.P. and Emmons, H.W., "Natural Convection above Fires," Journal of Fluid Mechanics, Vol. 8, 1960, pp. 611-624.

25. Priestley, C.H.B., and Ball, F.K., "Continuous Convection from an Isolated Source of Heat," Quart. Jour. of Roy. Meteor. Soc., Vol. 81, No. 348, 1955, pp. 144-157.

26. Rahm, S. Lennart, and Cederwall, Klas, "Submarine Disposal of Sewage," Proc., XI Congress, International Assoc. for Hyd. Research, Leningrad, Sept. 1965, Paper 2.17.

27. Rawn, A.M., and Palmer, H. K., "Predetermining the Extent of a Sewage Field in Sea Water," Trans. ASCE, Vol. 94, 1930, p. 1036.

28. Rawn, A.M., Bowerman, F.R., and Brooks, N.H., "Diffusers for Disposal of Sewage in Sea Water," Trans. ASCE, Vol. 126, Part III, 1961, pp. 344-388.

29. Sharp, James J., "Physical Interpretation of Jet Dilution Parameters," J. San. Engrg. Div., ASCE, Vol. 94, No. SA1, Feb. 1968, pp. 55-64.

30. Townsend, A.A., "The Mechanism of Entrainment in Free Turbulent Flows," Journal of Fluid Mechanics, Vol. 26, pp. 689-715, December 1966.

31. Turner, J. S., "Jets and Plumes with Negative or Reversing Buoyancy," Journal of Fluid Mechanics, Vol. 26, pp. 779-792, December 1966.

B. General Treatises and Case Studies

32. Brooks, N. H., "Controlling Ocean Pollution," Engineering and Science, Calif. Inst. of Tech., March 1966, pp. 13-17.

33. Brooks, Norman H., "Man, Water, and Waste," Proc. of Conf. on Next Ninety Years, Calif. Inst. of Tech., June 1967. (Also reprinted with minor revisions in Technology Review, MIT, May 1968.)

34. Gunnerson, C.G., "Sewage Disposal in Santa Monica Bay, California," Trans. ASCE, Vol. 124, 1959, pp. 823-851.

35. Harremoes, Poul, "Prediction of Pollution from Planned Wastewater Outfalls," J. Water Pollution Control Federation, 38, Aug. 1966, pp. 1323-1333.

36. Lawrence, C. H., "Sanitary Considerations of Five-Mile Ocean Outfall," Trans. ASCE, Vol. 127, Part III, 1962, pp. 294-324.

37. Municipality of Metropolitan Seattle, "Disposal of Digested Sludge to Puget Sound, The Engineering and Water Quality Aspects," Seattle, Washington, July 1965.

38. Pearson, E.A., "An Investigation of the Efficacy of Submarine Outfall Disposal of Sewage and Sludge," State Water Pollution Control Board, Publ. 14, Sacramento, Calif., 1956.

39. Wiegel, Robert L., "Oceanographical Engineering," Prentice-Hall, 1964.

40. John Carollo Engineers (Lafayette, Calif.), Final Report on Design of Ocean Outfall No. 2 for County Sanitation Districts of Orange County, P.O. Box 8127, Fountain Valley, Calif. 1970 (in press).

C. Tables

41. U. S. Navy Hydrographic Office, "Tables for Sea Water Density," Publication No. 615, 1952.

N. H. Brooks, "Conceptual Design of Submarine Outfalls - I"

LIST OF FIGURE TITLES

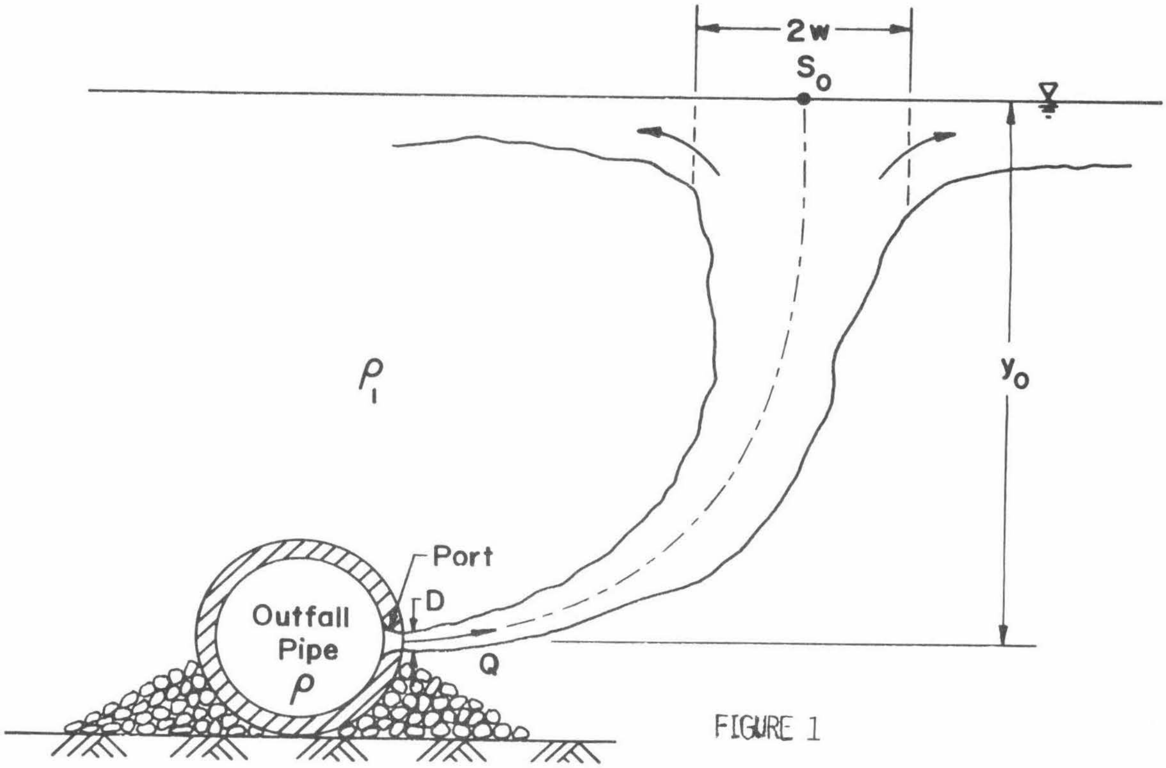
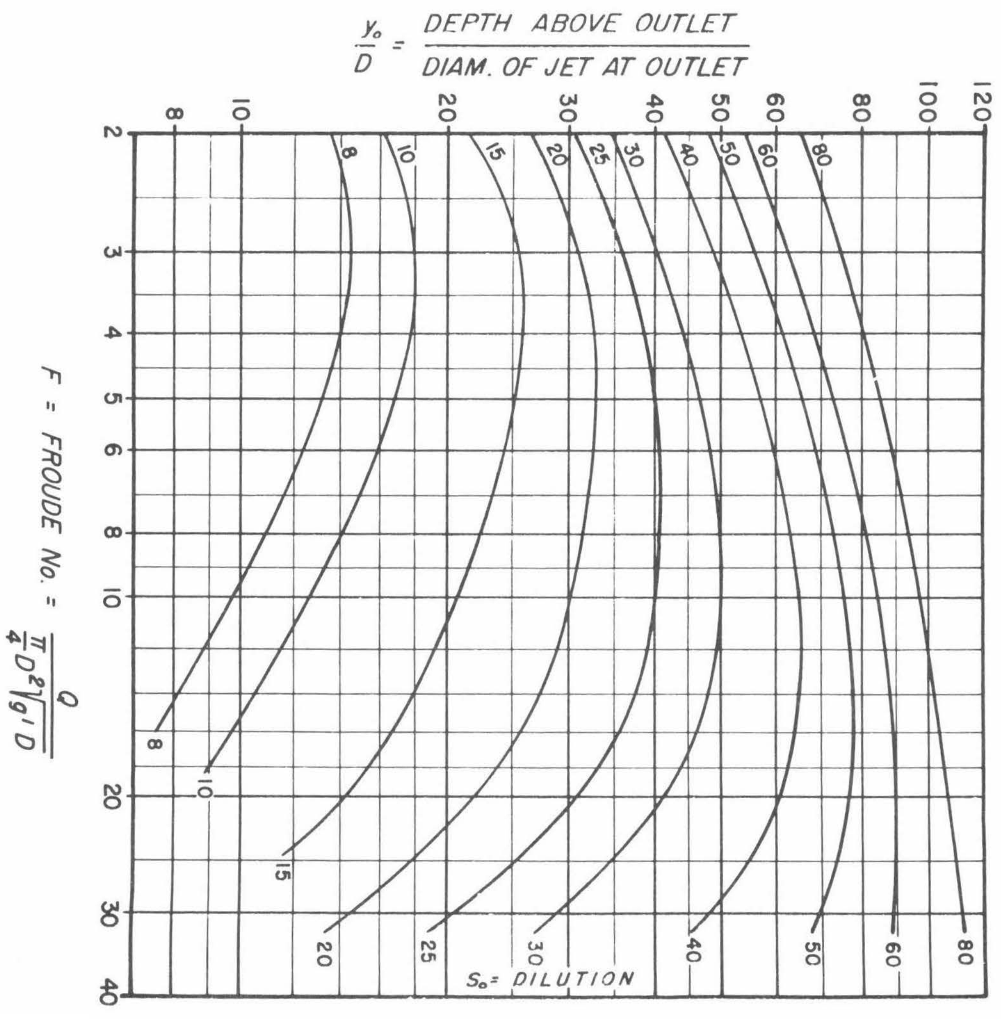

2w
S_o
ρ_1
y_o
Port
D
Outfall Pipe
ρ
Q

FIGURE 1


$\frac{y_o}{D}$ = DEPTH ABOVE OUTLET / DIAM. OF JET AT OUTLET
8 10 20 30 40 50 60 80 100 120
F = FROUDE No. = $\frac{Q}{\frac{\pi}{4}D^2\sqrt{g'D}}$
2 3 4 5 6 8 10 20 30 40
8 10 15 20 25 30 40 50 60 80
S_o = DILUTION

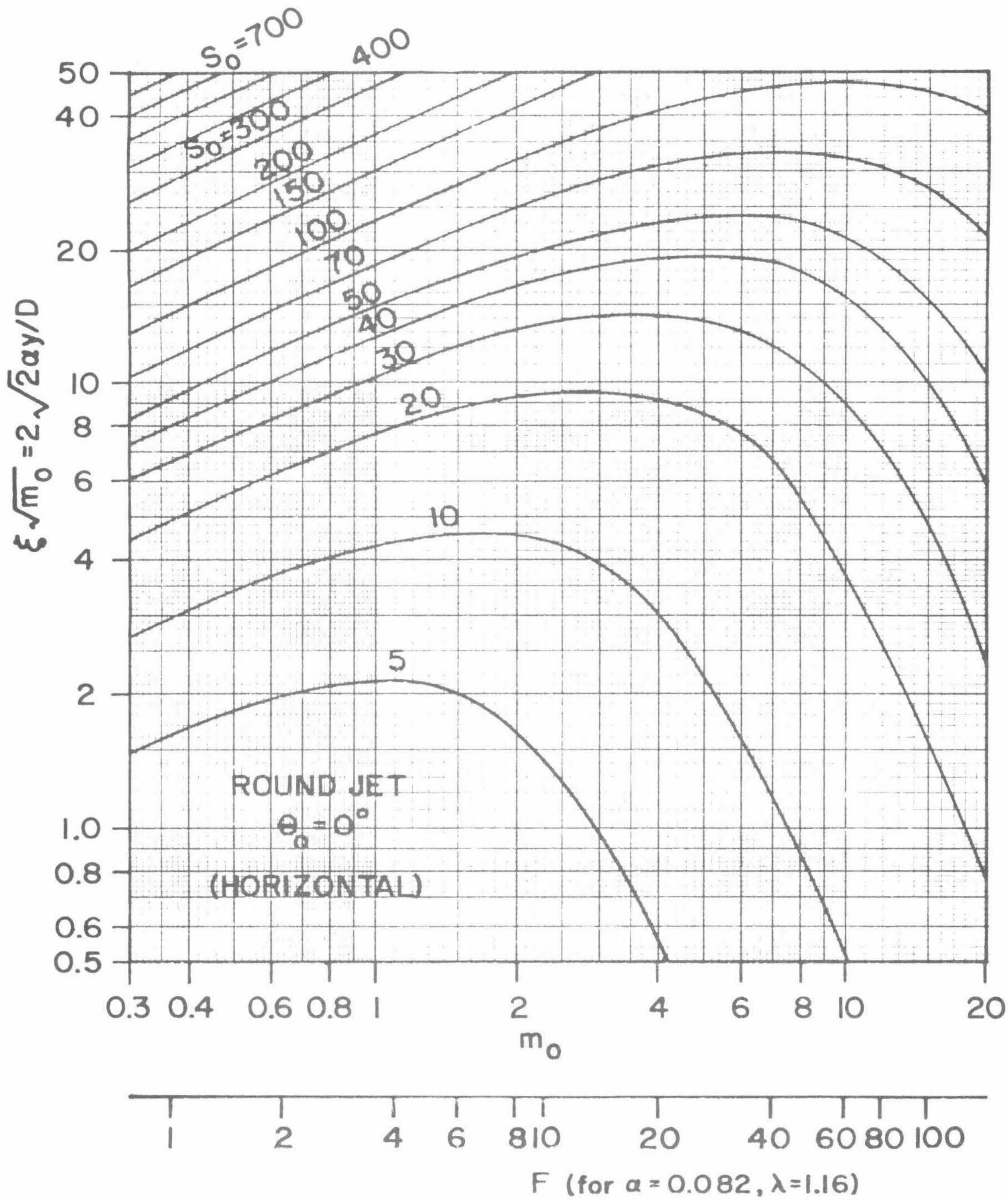
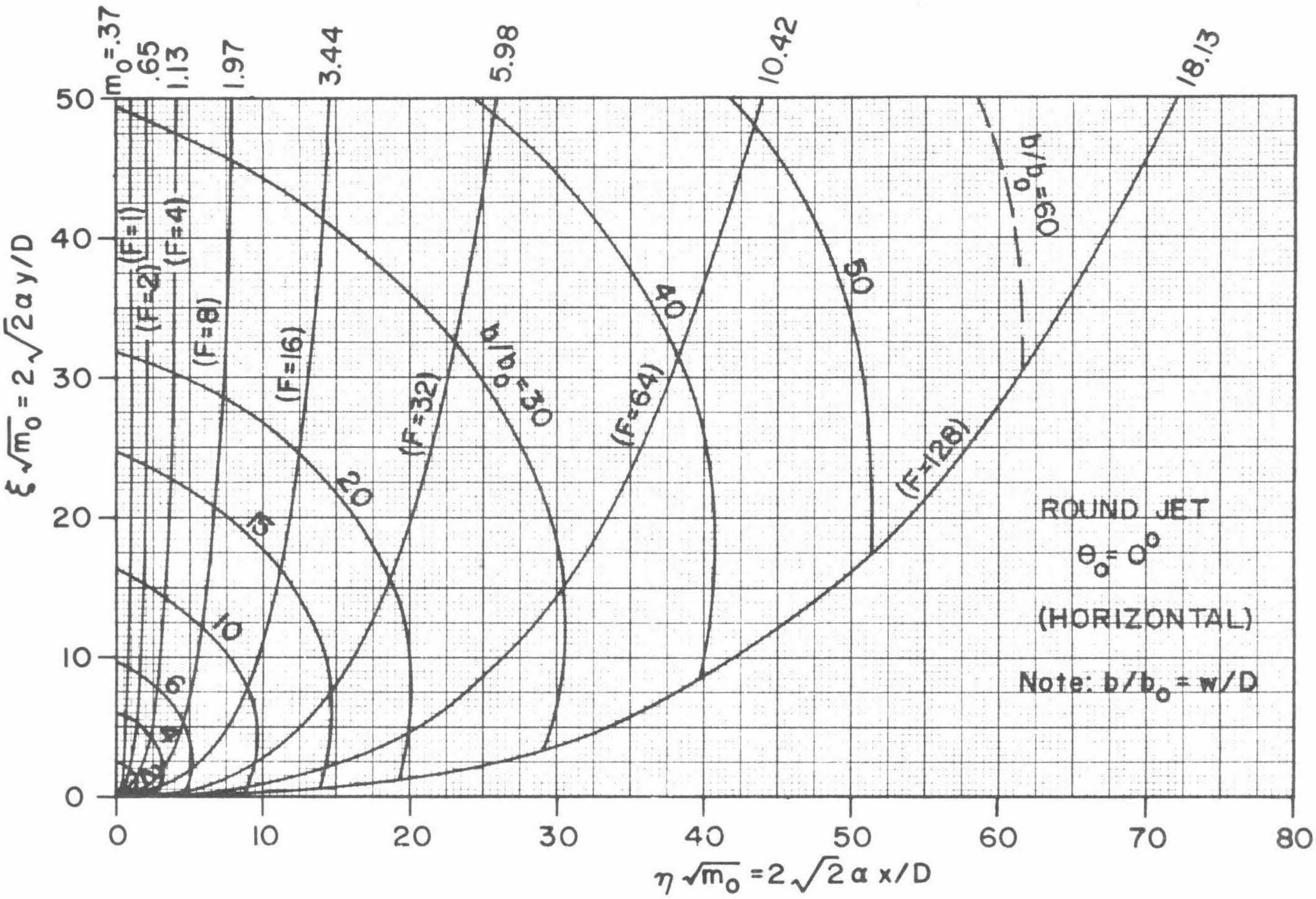

S0=700
400
S0=300
200
150
100
70
50
40
30
20
10
5
ROUND JET
θa = 0°
(HORIZONTAL)
ξ √m0 = 2√2αy/D
50
40
20
10
8
6
4
2
1.0
0.8
0.6
0.5
0.3 0.4 0.6 0.8 1 2 4 6 8 10 20
m0
1 2 4 6 8 10 20 40 60 80 100
F (for α = 0.082, λ = 1.16)

FIGURE 3



m_0=.37
.65
1.13
1.97
3.44
5.98
10.42
18.13
(F=1)
(F=2)
(F=4)
(F=8)
(F=16)
(F=32)
(F=64)
(F=128)
b/b_0=30
40
50
b/b_0=60
20
15
10
6
4
ROUND JET
$\theta_o = 0^o$
(HORIZONTAL)
Note: $b/b_o = w/D$
$\xi\sqrt{m_0} = 2\sqrt{2}\,\alpha\, y/D$
$\eta\sqrt{m_0} = 2\sqrt{2}\,\alpha\, x/D$
0
10
20
30
40
50
60
70
80

FIGURE 4

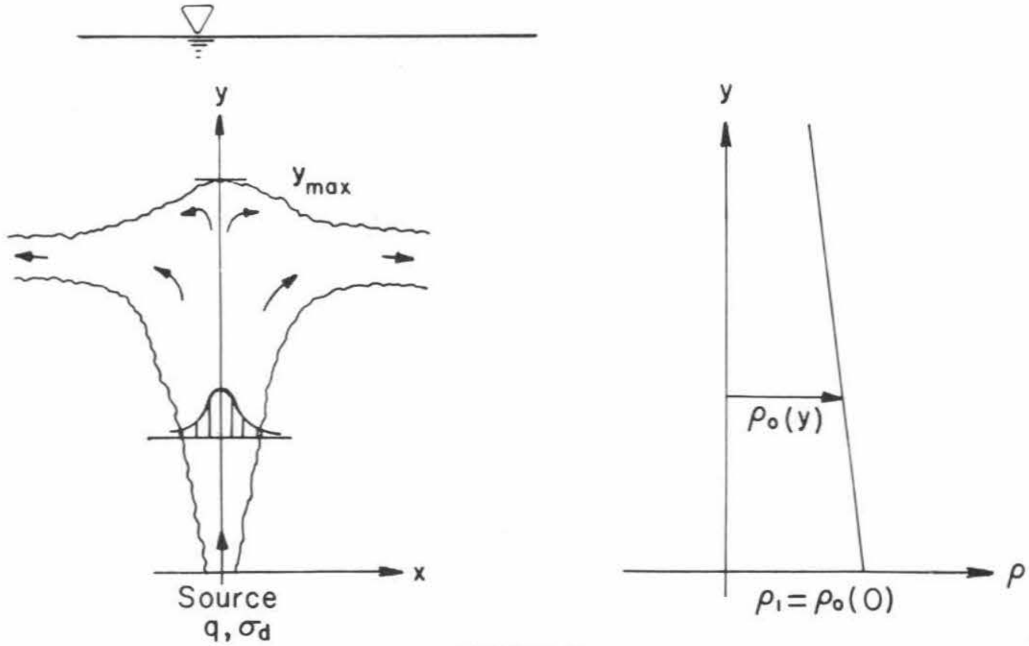


FIGURE 5

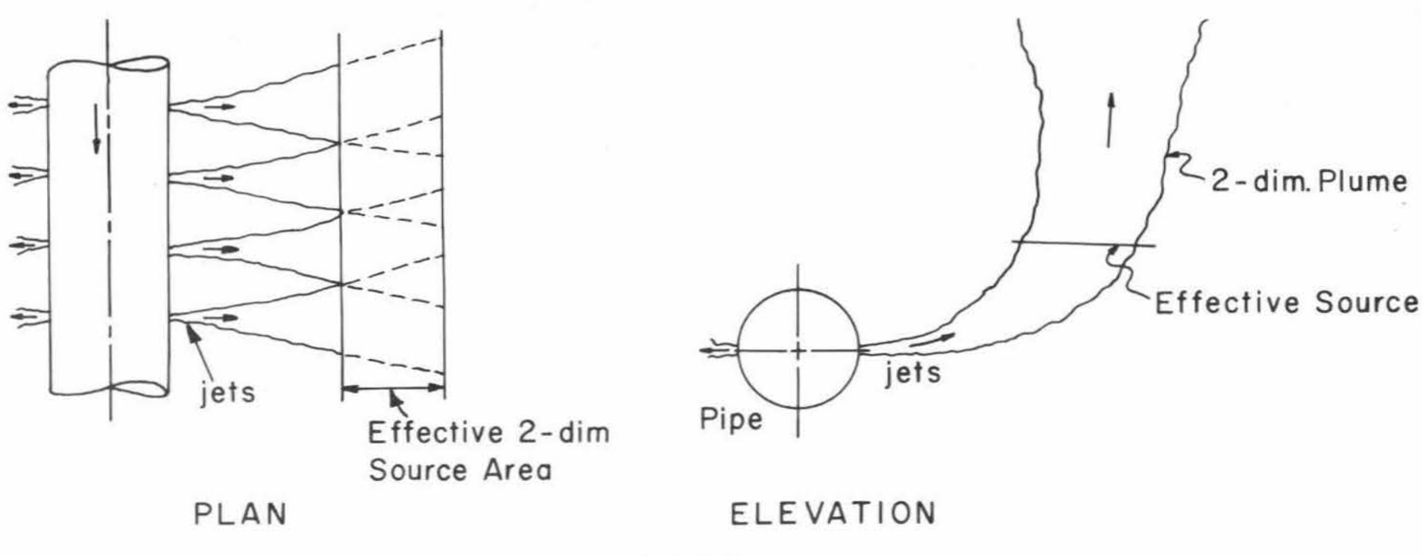


FIGURE 6

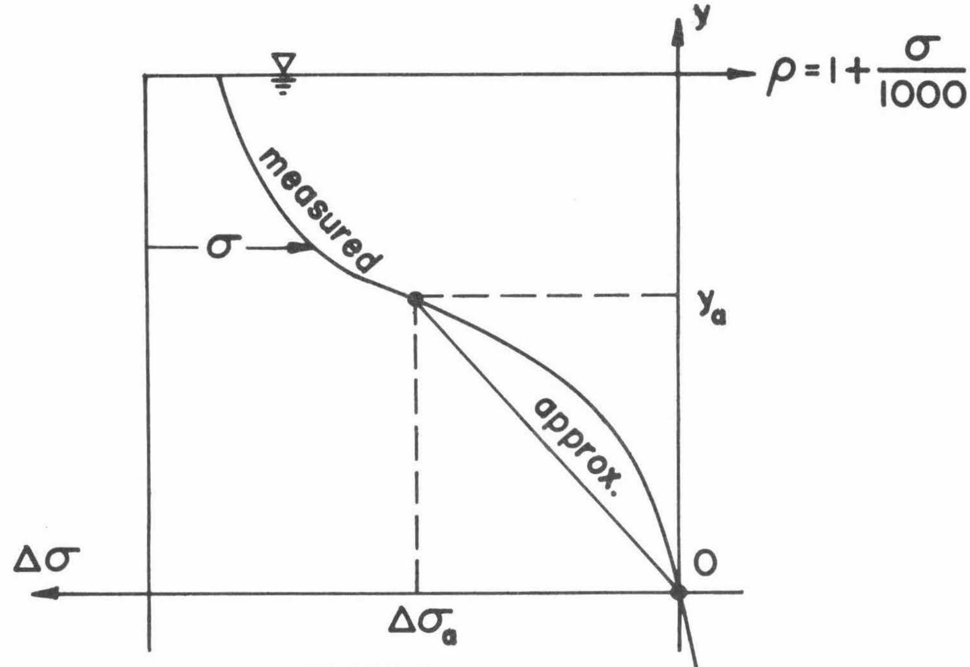


FIGURE 7

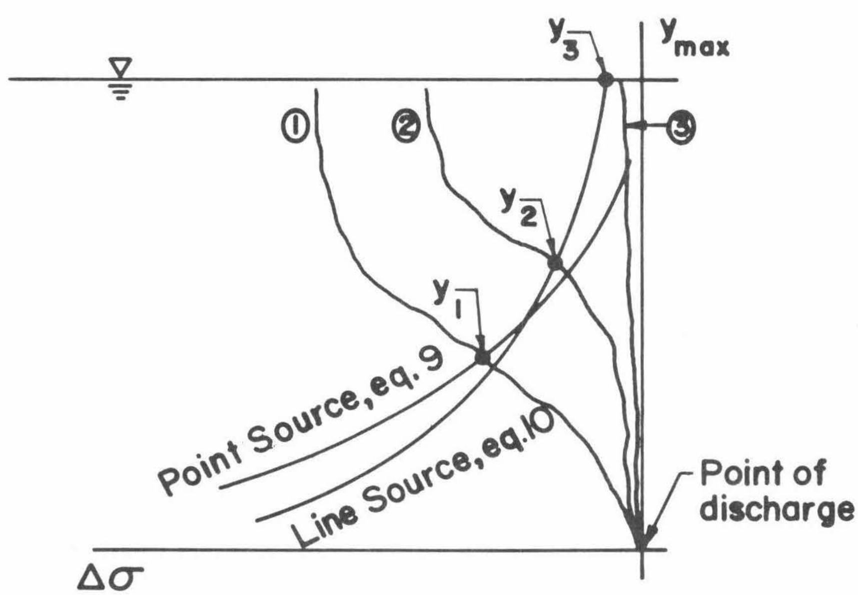


FIGURE 8